A SINGULAR VALUE OF Π.

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ON page 291 of Ray's Calculus may be seen a demonstration of the following well known theorem of Wallis:

$$\frac{\pi}{2}=\frac{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,6\,.\,8\,.\,8\,.\,.\,.\,.\,.}{1\,.\,3\,.\,3\,.\,5\,.\,5\,.\,7\,.\,7\,.\,9\,.\,.\,.\,.\,.} \tag{1}$$

By the binomial formula

$$(1-1)^n=1-n+\frac{n(n-1)}{2}-\frac{n(n-1)(n-2)}{2\,.\,3}+\frac{n(n-1)(n-2)(n-3)}{2\,.\,3\,.\,4}\ldots \tag{2}$$

Factoring,

$$(1-1)^n=\frac{(1-n)\;(2-n)\;(3-n)\;(4-n)}{1\;\;.\;\;2\;\;.\;\;3\;\;.\;\;4}\ldots \tag{3}$$

Substituting $-n$ for n,

$$(1-1)^{-n}=\frac{(1+n)\;(2+n)\;(3+n)\;(4+n)}{1\;\;.\;\;2\;\;.\;\;3\;\;.\;\;4}\ldots \tag{4}$$

Multiplying (3) by (4),

$$(1+1)^n(1-1)^{-n}=\frac{(1-n^2)(4-n^2)(9-n^2)(16-n^2)}{1\;\;.\;\;2\,.\,2\;\;.\;\;3\,.\,3\;\;.\;\;4\,.\,4}\ldots \tag{5}$$

Substituting $\frac{1}{2}$ for n, and reducing,

$$(1-1)^{\frac{1}{2}}(1-1)^{-\frac{1}{2}}=\frac{1\,.\,3\,.\,3\,.\,5\,.\,5\,.\,7\,.\,7\,.\,9\ldots}{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,6\,.\,8\,.\,8\ldots} \tag{6}$$

Combining (1) and (6),

$$\pi=\frac{2}{(1-1)^{\frac{1}{2}}(1-1)^{-\frac{1}{2}}}.$$

ANSWER TO PROF. SCHEFFER'S QUER (P. 31, VOL. VIII.)

BY C. B. SEYMOUR, ATTORNEY AT LAW, LOUISVILLE, KY.

Query.—"If of any curve we find the evolute, and of the latter the evolute, and so on ad infin., the ultimate evolute is a cycloid. How is this proved?"

Answer.—The proposition stated is not correct.

Let s_0 be the length of the given curve, measured from the origin to any point (the origin being a point on the curve). Let β_0 be the inclination of the tangent at that point to the axis of abscissas, and let R_0 be the radius of curvature at that point. Let s_n, β_n, R_n be like quantities for the corres-